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Higher loop corrections in noncommutative supersymmetric QED

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Abstract

The superfield formulation of the theory is systematically used to demonstrate the absence of quadratic divergences in the effective action up to two-loop order. The one-loop corrections to the gauge superfield three-point function is studied in detail in the covariant as well as in the Wess-Zumino gauge. We make some observations concerning the one-loop corrections of higher-point functions of the gauge superfield.

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During last years noncommutative (NC) field theories have been intensively studied. These theories emerged as the low energy limit of the open superstring in the presence of an external magnetic field (B-field) [1] although nowadays they are interesting in their own right (for a review see [2, 3]).

The most striking property of noncommutative field theories is undoubtly the UV/IR mechanism, through which the ultraviolet divergences (UV) in the commutative version of the theory are partly converted into infrared (IR) divergences [4, 5, 6]. These infrared divergences may be so severe that the perturbative expansion of the theory becomes meaningless. Hence, the key point about a noncommutative field theory is to determine whether it is renormalizable or not.

So far, only one four-dimensional noncommutative theory is known to be renormalizable, the Wess-Zumino model [7, 8]. In this case supersymmetry plays an essential role because it improves the ultraviolet behavior and, therefore, the UV/IR mechanism only generates mild infrared divergences which do not spoil the renormalization program. In three space-time dimensions we are aware of at least two noncommutative renormalizable models: the supersymmetric $O(N)$ nonlinear sigma model [9] and the $O(N)$ supersymmetric linear sigma model in the limit $N \to \infty$ [10]. As for nonsupersymmetric gauge theories the UV/IR mechanism breaks down the perturbative approach [5, 6, 11, 12, 13, 14, 15]. Nevertheless, we can entertain the hope that noncommutative supersymmetric gauge theories are still renormalizable and free from nonintegrable infrared singularities. We are aware of the following results concerning one-loop corrections in noncommutative supersymmetric gauge field theories:

1) In the work by Matusis and collaborators [5] the one-loop contributions to the two and three point functions of the gauge field component were studied with the conclusion that the leading divergences (both ultraviolet and infrared) canceled among themselves.

2) Zanon and collaborators [16, 17] used the background field method to calculate the one-loop contributions to the two point functions in N=1 and N=2 supersymmetric Yang Mills theories, where only logarithmic divergences were found. The three-point function was shown to vanish. For N=4 she demonstrated that, up to one loop, there are no divergences at all.

3) Bichl et al. [18] calculated the contributions to the two-point function of the gauge superfield and found cancellation of the quadratic divergences in the Feynman gauge.

In this work we deal with noncommutative supersymmetric QED without matter fields, in four space-time dimensions, within the framework of the superfield formulation. We demonstrate the absence of quadratic divergences in the effective action up to two-loop order. We also discuss in detail the one-loop contributions to the gauge superfield three-point functions in both the covariant Feynman gauge and the Wess-Zumino gauge. The work ends with some remarks concerning the singularity structure of higher loop corrections to the gauge superfield Green functions.

The classical action for the free supersymmetric QED (with Feynman type gauge fixing term) is [19]

$$
\begin{aligned}
S &= -\frac{1}{2g^2}\int d^8z(e^{-gV} * D^\alpha e^{gV})\bar{D}^2(e^{-gV} * D_\alpha e^{gV}) + \frac{1}{g^2}\int d^8z V\{D^2, \bar{D}^2\}V + \\
&+ \int d^8z(c+\bar{c})L_{gV/2}\left[-(c'+\bar{c}') + \coth L_{gV/2}(\bar{c}'-c')\right], \qquad (1)
\end{aligned}
$$

where V and $c, \bar{c}, c', \bar{c}'$ are the gauge and ghosts fields, respectively, $L_A B = [A, B]$ is the Lie derivative, D, $\bar{D}$ are the standard supercovariant derivatives [19] and all field products are to be understood as Moyal products. As pointed out above, one-loop corrections to the gauge superfield two-point function, deriving from this action, were already computed [18]so that we just summarize the results. The involved graphs are depicted in Fig.1. The first three diagrams are, individually, quadratically divergent whereas the last one is logarithmically divergent. One can verify that quadratic and linear divergences both cancel among themselves leaving only logarithmic divergences. For instance, the leading (quadratically divergent) contributions from the first, second and third graphs are, respectively, given by

$$
\begin{aligned}
S_1 &= \frac{2}{3}\int d^4\theta \int \frac{d^4p}{(2\pi)^4} V(-p,\theta)V(p,\theta)\int \frac{d^4k}{(2\pi)^4}\frac{\sin^2(k\wedge p)}{k^2}, \\
S_2 &= -\frac{1}{3}\int d^4\theta \int \frac{d^4p}{(2\pi)^4} V(-p,\theta)V(p,\theta)\int \frac{d^4k}{(2\pi)^4}\frac{\sin^2(k\wedge p)}{k^2}, \\
S_3 &= -\frac{1}{3}\int d^4\theta \int \frac{d^4p}{(2\pi)^4} V(-p,\theta)V(p,\theta)\int \frac{d^4k}{(2\pi)^4}\frac{\sin^2(k\wedge p)}{k^2}, \qquad (2)
\end{aligned}
$$

from which the cancellation of the quadratic divergences follows. Here $a \wedge b \equiv \frac{1}{2}a_\mu \theta^{\mu\nu} b_\nu$ and $\theta^{\mu\nu}$ is the antisymmetric constant matrix characterizing the underlying noncommutativity.

We recall next the background field formulation for the same model. The corresponding action reads [19] (see also [20] for a discussion of the background field method in superfield theories)

$$S = S_0 + S_{int} \tag{3}$$

where

$$\begin{aligned} S_0 &= -\frac{1}{2}\int d^8z V * (\Box - iW^\alpha\nabla_\alpha - i\bar{W}^{\dot\alpha}\bar\nabla_{\dot\alpha}) * V + \int d^8z(\bar{c}'c - c'\bar{c}) \qquad (4)\\ S_{int} &= \int d^8z\Big(\frac{1}{2}gV * \{\nabla^\alpha V, \bar\nabla^2\nabla_\alpha V\} - \frac{2}{3}g[[\nabla^\alpha V, V], V] * iW_\alpha - \\ &- \frac{1}{4}g^2[V, \nabla^\alpha V] * \bar\nabla^2[V, \nabla_\alpha V] - \frac{1}{3}g^2\bar\nabla^2\nabla_\alpha V * [V, [V, \nabla_\alpha V]] - \\ &- \frac{1}{6}g^2[[[\nabla^\alpha V, V], V], V] * iW_\alpha\Big) + \\ &+ \int d^8z\Big(-\frac{1}{2}g(c' + \bar{c}') * [V, c + \bar{c}] + \frac{1}{12}g^2(c + \bar{c}) * [V, [V, \bar{c}' - c']]\Big) + \ldots, \qquad (5) \end{aligned}$$

where W^α is the background field strength, $\nabla_\alpha, \bar\nabla_{\dot\alpha}$ are the background covariant derivatives whose action on an arbitrary superfield A is defined by

$$\begin{aligned} W_\alpha &= \frac{1}{2}[\bar\nabla^{\dot\alpha}, \{\bar\nabla_{\dot\alpha}, \nabla_\alpha\}]; \\ \nabla_\alpha A &= e^{-g\Omega}D_\alpha(e^{g\Omega}A),\ \bar\nabla_{\dot\alpha} = e^{g\bar\Omega}\bar{D}_{\dot\alpha}(e^{-g\bar\Omega}A), \qquad (6) \end{aligned}$$

where Ω and $\bar\Omega$ are background fields defined by the splitting of the quantum V gauge field [21]

$$e^{gV} \to e^{g\Omega}e^{gV}e^{g\bar\Omega}\,. \tag{7}$$

The ghosts c, c' are background covariantly chiral superfields, and the ghosts $\bar{c}, \bar{c'}$ are background covariantly antichiral superfields, i.e. the following conditions are satisfied:

$$\nabla_\alpha\bar{c} = \nabla_\alpha\bar{c'} = 0;\ \bar\nabla_{\dot\alpha}c = \bar\nabla_{\dot\alpha}c' = 0 \tag{8}$$

The free superpropagators arising from (4) are

$$\begin{aligned} &< \bar{c}c' > = < \bar{c}'c > = -(\Box - iW^\alpha\nabla_\alpha - \frac{i}{2}(\nabla^\alpha W_\alpha))^{-1}\delta^8(z - z')\,, \\ &< VV > = (\Box - iW^\alpha\nabla_\alpha - i\bar{W}^{\dot\alpha}\bar\nabla_{\dot\alpha})^{-1}\delta^8(z - z')\,, \qquad (9) \end{aligned}$$

but in the actual calculations we will always expand them into powers series of $W^\alpha, \bar{W}^{\dot\alpha}$. In this way we fix the degree of superficial divergence for a generic graph as

$$\omega = 2 - \frac{3}{2}N_W - \frac{1}{2}N_\nabla + \epsilon\,. \tag{10}$$

Here N_W is the number of external field strength lines, N_∇ is the number of spinor supercovariant derivatives associated with external W-lines, $\epsilon = 1$ for chiral contributions and zero otherwise.

A point worth mentioning is that within the framework of the background field method the only external lines are $W^\alpha, \bar{W}^{\dot{\alpha}}$ and their supercovariant derivatives; there are no external V fields. Contributions with three or more external $W, \bar{W}$ lines are superficially convergent, and graphs with two external $W, \bar{W}$ lines are at most superficially logarithmically divergent. This does not preclude that they can have subgraphs (with only external V and ghost lines) that by (10) can be quadratically or linearly divergent. The background field strength factors, W and $\bar{W}$, associated with a given supergraph, are not only provided by the vertices in (5) and the expansion of the propagators in (9) (which we will call henceforth explicit lines) but may also arise through the rule [19, 20]

$$[\nabla_\alpha, \Box] = \bar{W}^{\dot{\alpha}} \nabla_{\alpha\dot{\alpha}} + \frac{1}{2}(\nabla_{\alpha\dot{\alpha}} \bar{W}^{\dot{\alpha}}) \,. \tag{11}$$

It will also prove useful to recall the Bianchi identity [19, 20],

$$i W^\alpha \nabla_{\alpha\dot{\alpha}} \bar{W}^{\dot{\alpha}} = W^\alpha D^2 W_\alpha \,, \tag{12}$$

which allows for the conversion of factors $\bar{W}^{\dot{\alpha}}$ into factors W^α. We end this brief summary about the background field formulation by noticing that, as seen from Eq.(5), any vertex that is not associated with $W^\alpha, \bar{W}^{\dot{\alpha}}$ carries two ∇'s and two $\bar{\nabla}$'s, while the vertex associated with one $W^\alpha(\bar{W}^{\dot{\alpha}})$ carries only one ∇ ($\bar{\nabla}$) factor. Contraction of a loop into a point, in θ-space, via the rule [19, 20]

$$\delta_{12} \nabla^2 \bar{\nabla}^2 \delta_{12} = \delta_{12} \tag{13}$$

requires just two ∇ and two $\bar{\nabla}$ factors.

The one loop corrections to the two-point function were already found in [16]. They are proportional to W^2 and contain only logarithmic divergences. We turn now into computing the two-loop corrections to the two point function. The more general structures of the two-loops supergraphs to be considered are those in Fig.2.

In the absence of explicit lines, a one-vertex graph contains two ∇ and two $\bar{\nabla}$ factors while a two-vertex graph contains four ∇ and four $\bar{\nabla}$ factors. Hence, according to the rule in Eq.(13), for contracting loops into points in θ-space, there are no factors left to form W^2.

What come next are the supergraphs with one explicit W line. The only supergraph with enough ∇-factors to contract both loops into points, in θ-space, is that in Fig.3. This supergraph contains four ∇-factors in each vertex involving three internal V-lines (see (5)) and one ∇-factor in the vertex involving the explicit W^α-line. Contributions of the first order in W^α turn out to be proportional to $\int d^4\theta \nabla^\alpha W_\alpha$ and, therefore, vanish for being the integral of a total derivative. Contributions proportional to W^2 can be obtained by using the expression

$$\frac{1}{\Box}W^\alpha \nabla_\alpha \frac{1}{\Box} = \frac{1}{\Box}W^\alpha \frac{1}{\Box}\nabla_\alpha - \frac{1}{\Box}W^\alpha \frac{1}{\Box}(\frac{1}{2}(\nabla_{\alpha\dot\alpha}\bar W^{\dot\alpha}) + \bar W^{\dot\alpha}\nabla_{\alpha\dot\alpha})\frac{1}{\Box} \tag{14}$$

which follows from (11). We remark that each additional field strength factor $\bar W_{\dot\alpha}$ is, correspondingly, accompanied by an additional $\Box^{-1}$ factor.

To get the contribution proportional to W^2 we move the ∇_α to the nearest vertex using the relation (14). The term with only one explicit field strength will not contribute, since it is proportional the $\delta_{12}\nabla^3\bar\nabla^2\delta_{12}\nabla^2\bar\nabla^2\delta_{12} = 0$. However, there are other two contributions each with one W^α, already present from the beginning, and one $\bar W^{\dot\alpha}$ that is generated through (14). The first of these two terms is proportional to $\int d^8z W^\alpha \frac{1}{\Box}\nabla_{\alpha\dot\alpha}\bar W^{\dot\alpha}$ which, with the help of (12), can be cast as $\int d^8z W^\alpha \frac{D^2}{\Box} W_\alpha$ and, when represented as an integral over the chiral subspace, takes the form $\int d^6z W^2$. The other term, which is of the form $W^\alpha \frac{1}{\Box}\bar W^{\dot\alpha}\nabla_{\alpha\dot\alpha}$, after integration over the loop momenta becomes proportional to $\int d^8z W^\alpha \frac{1}{\Box}\nabla_{\alpha\dot\alpha}\bar W^{\dot\alpha}$ and exhibits the same structure. To summarize, the contributions of Fig.3 are proportional to W^2 and, therefore, contain at most logarithmic subdivergences.

What remains to be done is to study the supergraphs with two explicit W lines. To isolate the divergent contributions proportional to W^2 we follow the procedure designed by Zanon and collaborators [16]. Hence, we shall be considering diagrams with two explicit W^α lines where one of the vertices, $V*(-iW^\alpha\nabla_\alpha)*V$, has been replaced by $V*(-iW^\alpha D_\alpha)*V$ while the other is substituted by $V*(-i\Gamma^\beta \bar D^2 D_\beta)*V + \ldots$. Here, Γ^β is a connection satisfying the condition $\bar D^2\Gamma^\beta = W^\beta$. There are other terms containing less spinor derivatives which however are not enough to contract a loop into a point or to produce W^2. Therefore they do not give divergences proportional to W^2.

The two-loop supergraphs containing nonvanishing quadratic subdivergences are depicted in Fig. 4. They can be thought as arising from the first three diagrams in Fig.1 after closing the external lines into a propagator and then inserting the explicit field strength lines.

Therefore, the sum of the quadratically divergent parts of the two-loop supergraphs in Fig.4 also vanishes. There are several two-loop supergraphs whose contributions are proportional to $\nabla^\alpha \nabla^2 = 0$.

The remaining nontrivial two-loop supergraphs are those in Fig.5. The amplitudes associated with these diagrams are found to read, respectively,

$$
\begin{aligned}
&\int d^4\theta \int \frac{d^4k d^4l d^4p}{(2\pi)^{12}} \frac{\Delta_1(k,l,p)}{k^2 l^2 (k+l+p)^2 (l+p)^2} W^\alpha(p) W_\alpha(-p)\,, \\
&\int d^4\theta \int \frac{d^4k d^4l d^4p}{(2\pi)^{12}} \frac{\Delta_2(k,l,p)}{k^2 l^2 (k+p)^2 (l+p)^2} W^\alpha(p) W_\alpha(-p)\,, \\
&\int d^4\theta \int \frac{d^4k d^4l d^4p}{(2\pi)^{12}} \frac{\Delta_3(k,l,p)(k\cdot l + l^2)}{k^2 l^2 (k+l)^2 (k+l+p)^2 (l+p)^2} W^\alpha(p) W_\alpha(-p)\,, \\
&\int d^4\theta \int \frac{d^4k d^4l d^4p}{(2\pi)^{12}} \frac{\Delta_4(k,l,p)(k\cdot l + l^2)}{k^2 l^2 (k+l)^4 (k+l+p)^2} W^\alpha(p) W_\alpha(-p)\,,
\end{aligned}
\tag{15}
$$

where $\Delta_i(k,l,p), i = 1,2,3,4$ are phase factors. Straightforward power counting reveals that all divergences are logarithmic.

Since there are no other supergraphs giving rise to divergent contributions to the effective action, we conclude that the two-loop counterterm in the theory is proportional to W^2 and it must cancels a logarithmic divergence. Moreover, within the framework of the background field method, the UV/IR mixing mechanism only gives rise to a harmless logarithmic IR divergence.

Our next purpose in this work is to study higher-point one-loop functions with external V lines. These diagrams are necessarily to be considered as subgraphs of larger diagrams. In this connection, we recall, once more, that in the background field method the external lines can only be associated with superfield strengths and their supercovariant derivatives.

We shall look first for the one-loop three point function. The expansion of the action (5) must, then, include terms up to the fifth order in the quantum fields. The terms of third, fourth and fifth order, in the field V, are found to read, respectively,

$$
S_3 = \int d^8z \Big(g\bar{D}^2 D^\alpha V * [V, D_\alpha V] - \frac{ig}{2}(c+\bar{c})[V, \bar{c}' + c'] \Big)\,, \tag{16}
$$

$$
\begin{aligned}
S_4 &= \int d^8z \Big(-\frac{g^2}{4}[V, D^\alpha V] * \bar{D}^2 [V, D_\alpha V] - \frac{g^2}{3}\bar{D}^2 D^\alpha V * [V,[V, D_\alpha V]] \\
&+ \frac{ig^2}{12}(c+\bar{c}) * [V,[V,\bar{c}' - c']] \Big)\,, \\
S_5 &= \frac{g^3}{24}\int d^8z \Big(\bar{D}^2 D^\alpha V * [V,[V,[V, D_\alpha V]]] + 2[V, \bar{D}^2 D^\alpha V] * [V,[V, D_\alpha V]] \Big)\,.
\end{aligned}
\tag{17}
$$

Let us first look at the three-point functions with external V fields (which appear as subgraphs in the effective action) but not with external background field strengths. In this case we are allowed to use "flat" supercovariant derivatives. All commutators are, of course, Moyal ones. We emphasize that there is no ghost vertex of third order in V because $(c+\bar{c})L_{gV}(\coth L_{gV}(\bar{c}'-c'))$ only contains even powers in V. Contraction of any loop into a point requires, as usual, two D and two $\bar{D}$ factors. Furthermore, the vertex S_5 can be casted as

$$
\begin{aligned}
S_5 \;=\; & g^3 \int d^4\theta \int \frac{d^4k_1 \ldots d^4k_5}{(2\pi)^{20}} (2\pi)^4 \delta(k_1+\ldots+k_5) \bar{D}^2 D^\alpha V(k_1) V(k_2) V(k_3) V(k_4) D_\alpha V(k_5) \\
\times \; & \Big\{ -\frac{i}{3} \sin(k_1 \wedge k_2) [\cos(k_3 \wedge k_4) \cos((k_3+k_4) \wedge k_5) - \cos(k_3 \wedge k_4 + k_3 \wedge k_5 - k_4 \wedge k_5)] \\
+ \; & \frac{i}{12} [\cos(k_3 \wedge k_4) \sin(k_1 \wedge k_2 + (k_3+k_4) \wedge k_5) \\
+ \; & 3 \sin(k_5 \wedge k_3) \cos(k_1 \wedge k_2 + k_4 \wedge k_1 - k_2 \wedge k_4)] \Big\} . \qquad (18)
\end{aligned}
$$

Since the vertex cV^3c is absent (see observation above), the contributions to the three-point function coming from the ghost sector are reduced to the diagrams in Fig.6. The second diagram in Fig.6 just vanishes due to Furry theorem. Let us now analyze the first diagram. To start with, we isolate the momentum integral whose integrand is, up to irrelevant constants, $< c\bar{c}' >< \bar{c}'c > - < \bar{c}c' >< c'\bar{c} >$. These correlators originate from contractions linking the vertices $c[V,\bar{c}']$ with $c[V,[V,\bar{c}']]$ and $\bar{c}[V,c']$ with $-\bar{c}[V,[V,c']]$. Each product of correlators give rise to a quadratically divergent integral but when combined, as indicated, these integrals cancel out between themselves,

$$
\begin{aligned}
& \frac{g^3}{12} V_m^{(3)}(k, k+p_1+p_2, -p_1-p_2) V_m^{(4)}(k, p_1, p_2, -k-p_1-p_2) \Big(\frac{D^2\bar{D}^2}{k^2} \delta_{12} \times \\
\times \; & \frac{\bar{D}^2 D^2}{(k+p_1+p_2)^2} \delta_{12} - \frac{\bar{D}^2 D^2}{k^2} \delta_{12} \frac{D^2 \bar{D}^2}{(k+p_1+p_2)^2} \delta_{12} \Big) = 0 \, . \qquad (19)
\end{aligned}
$$

Here, to simplify the notation, we have introduced the definitions
$V_m^{(3)}(k_1,k_2,k_3) = i \sin(k_1 \wedge k_2)$ whereas
$V_m^{(4)}(k_1,k_2,k_3,k_4) = \cos(k_1 \wedge k_4)\cos(k_2 \wedge k_3) - \cos(k_1 \wedge k_2 + k_4 \wedge k_3)$. We focus next on the contraction of $c[V,c']$ with $\bar{c}[V,[V,\bar{c}']]$ and similar ones. They yield contributions in which spinor derivatives act on external lines and are, therefore, only logarithmically divergent.

All that remains to be considered are the contributions to the gauge superfield three-point function coming from gauge loops. They are displayed in Fig.7. The first graph in Fig.7 only contains logarithmic divergences, just as it happened in the case of the last diagram of Fig.1. From the point of view of divergences, the potentially dangerous terms in the amplitude associated with the second graph are those not involving derivatives acting on the V-field external lines. To extract the leading divergence, we start by recalling, once more, that there are four D-factors associated with a three V-line vertex. This amounts to a total number of twelve D-factors. Four of them are used to contract the loop into a point in θ-space. The remaining eight D-factors give origin to a term k^4, since $D^2\bar{D}^2D^2 = -k^2D^2$. Hence, in the case of the second graph of Fig.7, the leading divergence is quadratic and the corresponding piece of the amplitude is found to read

$$\begin{aligned}\Gamma_2 \ &= \ \frac{2}{3}ig^3\int\frac{d^4k}{(2\pi)^4}\frac{\sin(k\wedge p_1)\sin((k+p_1)\wedge p_2)\sin(k\wedge(p_1+p_2))}{k^2}\times \\ &\times \ V(-p_1)V(-p_2)V(-p_3)\,. \end{aligned} \tag{20}$$

As for the third graph in Fig.7, D-algebra transformations enable one to find, for the leading divergent term,

$$\begin{aligned}\Gamma_3 \ &= \ ig^3\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\Big(-\frac{1}{3}\sin(k\wedge p_1)[\cos(p_2\wedge p_3)\cos(k\wedge(p_2+p_3))- \\ &- \ \cos(p_2\wedge p_3+k\wedge(p_2-p_3))]+ \\ &+ \ \frac{1}{12}\cos(p_2\wedge p_3)\sin(k\wedge(p_2+p_3+p_4))-\frac{1}{4}\sin(k\wedge p_3)\cos(p_1\wedge p_3-k\wedge(p_1-p_3))\Big)\times \\ &\times \ V(-p_1)V(-p_2)V(-p_3)\,, \end{aligned} \tag{21}$$

also signalizing for the presence of quadratic divergences. After rearrangement of the trigonometric factors and taking into account the magic of dimensional regularization,

$$\int\frac{d^dk}{(2\pi)^4}\frac{1}{k^2}=0\,, \tag{22}$$

which leads to the vanishing of the planar part, one arrives at

$$\begin{aligned}\Gamma_2 \ &= \ -\frac{1}{6}ig^3\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\times \\ &\times \ \Big(\sin(2k\wedge(p_1+p_2)+p_1\wedge p_2)-\sin(2k\wedge p_1-p_1\wedge p_2)+\sin(2k\wedge p_2+p_1\wedge p_2)\Big)\times \\ &\times \ V(-p_1)V(-p_2)V(-p_3) \end{aligned} \tag{23}$$

and

$$\begin{aligned}\Gamma_3 \ &= \ ig^3\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\Big(\frac{1}{6}\sin(k\wedge(p_1-p_2+p_3)+p_2\wedge p_3)+ \\ &+ \ \frac{1}{6}\sin(k\wedge(p_1+p_2-p_3)-p_2\wedge p_3)-\frac{1}{8}[\sin(k\wedge(p_1+p_2-p_3)-p_1\wedge p_3)- \\ &- \ \sin(k\wedge(p_2+p_3-p_1)+p_1\wedge p_3)]\Big)\times \\ &\times \ V(-p_1)V(-p_2)V(-p_3)\,, \end{aligned} \tag{24}$$

where momentum conservation $p_1 + p_2 + p_3 = 0$ has been used where needed.

We emphasize that the net effect of (22) is to wash out all planar (UV) quadratically divergent terms from Γ_2 and Γ_3. A closed form for the remaining nonplanar parts can be obtained after performing the momentum integrals in (23) and (24) with the help of

$$\int \frac{d^4k}{(2\pi)^4 k^2} \cos(2k \wedge p) = \frac{1}{4\pi^2 p \circ p}\,, \tag{25}$$

where $p \circ p \equiv p^\mu (\theta)^2_{\mu\nu} p^\nu$. One obtains

$$\begin{aligned} \Gamma_2 &= -\frac{1}{96\pi^2} g^3 i \sin(p_1 \wedge p_2)\left(\frac{1}{p_1 \circ p_1} + \frac{1}{p_2 \circ p_2} + \frac{1}{p_3 \circ p_3}\right) \times \\ &\times\ V(-p_1)V(-p_2)V(-p_3) \end{aligned} \tag{26}$$

and

$$\Gamma_3 = -\frac{g^3}{384\pi^2} i \sin(p_1 \wedge p_3)\left(\frac{1}{p_1 \circ p_1} - \frac{1}{p_3 \circ p_3}\right) V(-p_1)V(-p_2)V(-p_3)\,. \tag{27}$$

After symmetrization $p_1 \to p_2 \to p_3$, Γ_3 vanishes. Hence, the only effective quadratic IR divergence left is that in Eq.(26).

Thus, in the one-loop approximation, the gauge superfield three-point function is plagued with IR quadratic divergences. This might appear to contradict the results in Ref.[5]. However, this is not the case. Indeed, from the expression for V in terms of field components, i.e.,

$$\begin{aligned} V(x,\theta) &= C(x) + \chi^\alpha(x)\theta_\alpha + \bar\chi_{\dot\alpha}(x)\theta^{\dot\alpha} + \theta^2 M(x) + \bar\theta^2 N(x) + (\bar\theta\sigma^m\theta)A_m(x) + \\ &+\ \theta^2\bar\theta_{\dot\alpha}\bar l^{\dot\alpha}(x) + \bar\theta^2\theta^\alpha l_\alpha + \theta^2\bar\theta^2 F(x)\,, \end{aligned} \tag{28}$$

one can easily see that the gauge superfield three-point function does not contain the three-point function of the gauge potential A_m as one of its components. In fact, the three-point photon function studied in [5] is one of the components of the superfield-three point function involving two derivatives of V and, therefore, being free of quadratic singularities. Moreover, the term proportional to $\int d^4\theta V^3$ depends not only on the gauge field A_m but also on the lower dimensional fields $(M, N, C, \chi, \bar\chi)$ which are known to vanish in Wess-Zumino gauge. This argument secures that higher-point functions of the V field being proportional to $\int d^4\theta V^n$, where n is the number of points, will also be free of quadratic divergences in the Wess-Zumino gauge. We emphasize that, within the framework of the background field method, n-point functions of V fields can only arise as subgraphs of graphs with n or higher number of loops and external $W, \bar{W}$ strength legs. In this last connection, the graphs given in Fig.7 could be subgraphs of three and higher loop supergraphs with external $W, \bar{W}$ legs.

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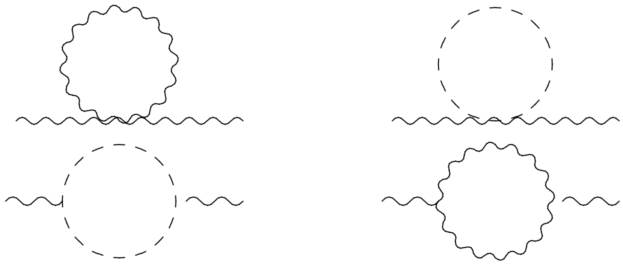

FIG. 1: Structure of one-loop two-point functions. Dashed and wavy lines represent, respectively, the propagators of the superghost and supergauge fields.

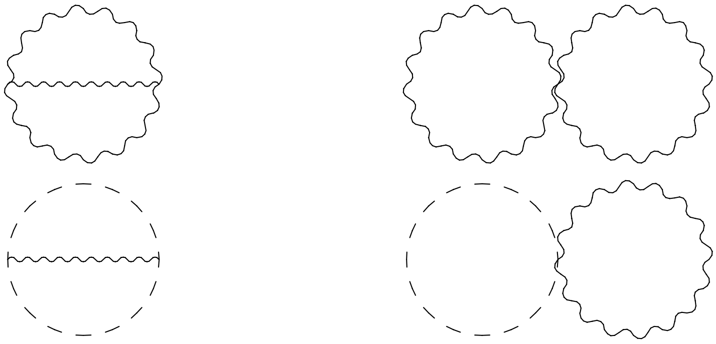

FIG. 2: General structure of two-loop supergraphs.

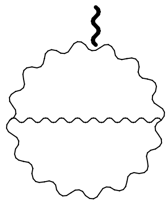

FIG. 3: Supergraph with one explicit field strength leg. The explicit background field strength is represented by heavy wavy line

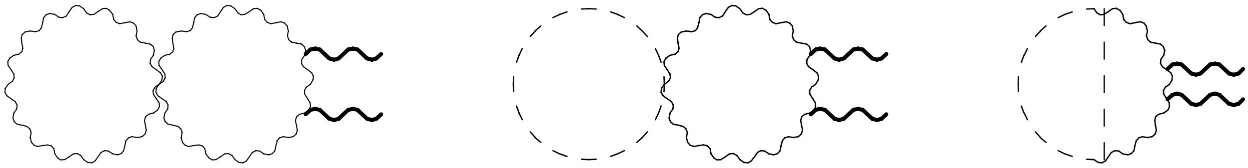

FIG. 4: Two-point quadratically divergent graphs proportional to W^2.

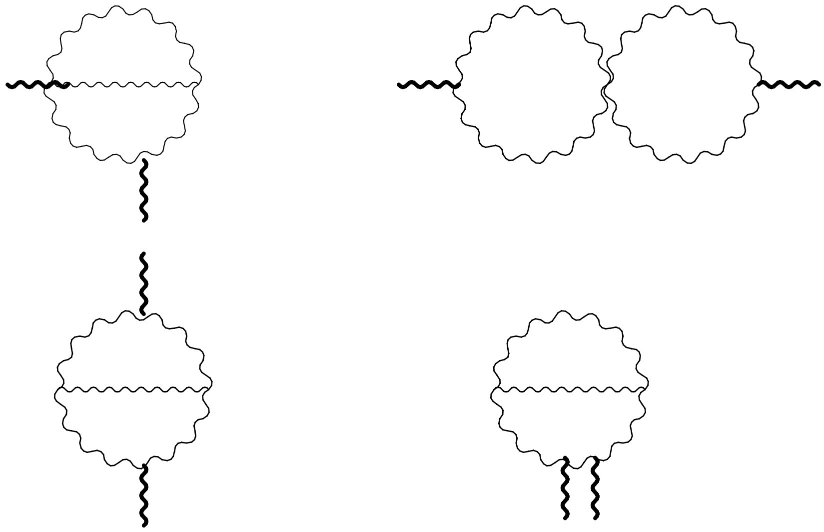

FIG. 5: Two-point logarithmically divergent graphs proportional to W^2

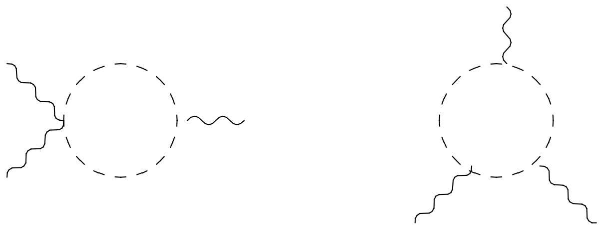

FIG. 6: Three-point functions originated from ghost sector.

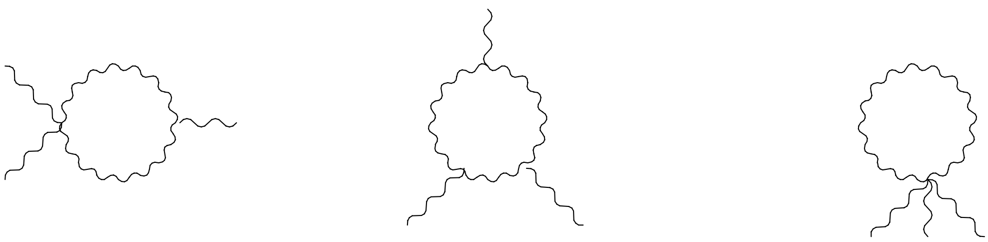

FIG. 7: Three-point functions originated from gauge sector.